TRINITY COLLEGE

IN NOMINE DOMINI

YEAR 12
MATHEMATICS
METHODS

Test 4, 2023
Section One: Calculator Free
Normal Distribution and Sampling

STUDENT'S NAME: Solutions [LAWRENCE]

DATE: Thursday 31st August **TIME:** 20 minutes **MARKS**: 23
ASSESSMENT %: 10

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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Question 1 **(6 marks)**

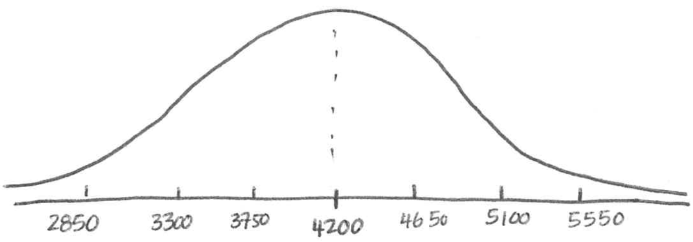
For a set of data values that are normally distributed, approximately 68% of the values will lie within one standard deviation of the mean, approximately 95% of the values will lie within two standard deviations of the mean and approximately 99.7% of the values will lie within three standard deviations of the mean.

If the weights of a large group of elephants are normally distributed with a mean of 4200 kg and a standard deviation of 450 kg, use the above information to answer the following questions:

a) A zoo keeper says that almost all of the elephants have weights in the range 2850 kg to 5550 kg. Comment on her statement. (2 marks)

99.7% of the weights lie within 2850 kg and 5550 kg

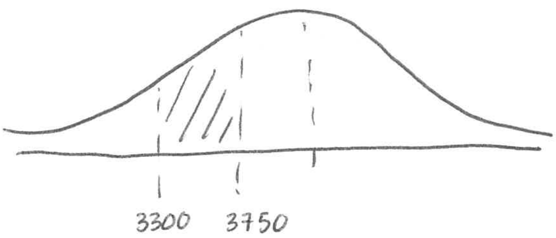
∴ she is correct



✓ stating she is correct

✓ using 99.7%

b) Approximately what percentage of elephants in the group has a weight between 3.3 tonnes and 3.75 tonnes. (2 marks)



95 − 68 = 27

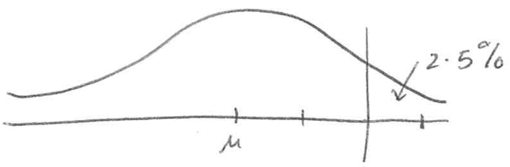
27 ÷ 2 = 13.5%

✓ use of 95 & 68

✓ splitting remainder into 2.

c) Approximately 2.5% of the elephants are heavier than what weight? (2 marks)

100 − 95 = 5 ÷ 2 = 2.5%



5100g

(5.1 tonnes)

✓ identifying correct region

✓ correct weight

Question 2 **(3 marks)**

A 90% confidence interval for a population proportion based on a sample size of 360 has width w.
What sample size is required to obtain a 90% confidence interval of width $\frac{w}{4}$?

$$W_2 = \frac{W_1}{4}$$

$$\cancel{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{\cancel{z}\sqrt{\frac{\hat{p}(1-\hat{p})}{360}}}{4}$$

✓ M of E statements
(1 with n, 1 with 360)

$$\sqrt{\frac{1}{n}} = \frac{\sqrt{\frac{1}{360}}}{4}$$

✓ using s.f of 4

$$\frac{16}{n} = \frac{1}{360}$$

✓ correct number of sample size.

$n = 16(360) = 5760$ sample size

Question 3 **(3 marks)**

In a Methods exam, the class achieved an average of 45% with a standard deviation of 15%. The teacher decided to scale the marks so that the mean would be 65% and the standard deviation 12%.

Jason got a raw score of 40%. What would be his scaled score?

$$Z_R = \frac{40-45}{15} = \frac{-5}{15} = -\frac{1}{3}$$ (Raw)

$$-\frac{1}{3} = \frac{x_s - 65}{12}$$ (Scaled)

$$-4 = x_s - 65$$

$$61 = x_s$$

✓ calculating raw z-score

✓ connecting with rule for scaled data

✓ correct scaled score

Question 4 **(11 marks)**

When calculating a confidence interval for a population proportion from a sample an associated z score is used.

Use the table below to answer the following questions:

Confidence Interval	z score (rounded to 1 decimal place)
95%	2.0
87%	1.5
68%	1.0

a) In a random sample of 100 people, 20 said they had watched an AFL game in the last year.

i) Determine the proportion of those in the sample who had watched an AFL game in the last year. (1 mark)

$$\hat{p} = \frac{20}{100} = 0.2$$

✓ correct $\hat{p}$

ii) Show that the standard deviation for the proportion is 0.04. (2 marks)

$$\sigma = \sqrt{\frac{0.2(0.8)}{100}} = \frac{\sqrt{0.16}}{10} = \frac{0.4}{10}$$

$$= 0.04$$

✓ correct use of σ rule

✓ correct simplification of surds.

iii) Determine a 95% confidence interval for the proportion of the population who had watched an AFL game in the last year. (2 marks)

$$0.2 \pm 2\sqrt{\frac{(0.2)(0.8)}{100}}$$

$$0.2 \pm 2(0.04)$$

$$0.2 \pm 0.08$$

$$0.12 \leq p \leq 0.28$$

✓ correct CI statement using $\hat{p}$ & σ from i) and ii)

✓ correct upper & lower boundary

A random sample size n_1 was taken and the proportion of people who had watched a game of AFL in the last year was m.

b) Determine a 68% confidence interval for the proportion of the population who had watched an AFL game in the last year in terms of n_1 and m. (2 marks)

$\hat{p} = m$
$n = n_1$
$z = 1$

$$m \pm 1\sqrt{\frac{m(1-m)}{n_1}}$$

✓ correct $\hat{p}$, n & z

✓ correct statement for CI

c) A new sample of size n_2 was taken and the proportion of people who had watched a game of AFL in the last year was again m. When an 87% confidence interval was determined it was found to be the same as the interval determined in part b).

i) Is n_2 larger or smaller than n_1? Explain. (2 marks)

$\hat{p} = m$
$n = n_2$
$z = 1.5$

$n_2 > n_1$

✓ larger

✓ valid reason (may refer to working as below).

ii) What is the relationship between n_1 and n_2? (2 marks)

$n_1 = \frac{4}{9} n_2$

$$1\sqrt{\frac{m(1-m)}{n_1}} = 1.5\sqrt{\frac{m(1-m)}{n_2}}$$

$$\sqrt{\frac{m(1-m)}{n_1}} = \frac{3}{2}\sqrt{\frac{m(1-m)}{n_2}}$$

$$\frac{1}{n_1} = \frac{9}{4}\frac{1}{n_2}$$

$$n_1 = \frac{4}{9} n_2$$

✓ correct scale factor

✓ valid working (as above)

END OF QUESTIONS

TRINITY COLLEGE

IN NOMINE DOMINI

YEAR 12
MATHEMATICS
METHODS

Test 4, 2023
Section Two: Calculator Allowed
Normal Distribution and Sampling

STUDENT'S NAME: Solutions [LAWRENCE]

DATE: Thursday 31st August **TIME:** 32 minutes **MARKS**: 36
ASSESSMENT %: 10

INSTRUCTIONS:

Standard Items:	Pens, pencils, drawing templates, eraser
Special Items:	1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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Question 5 **(3 marks)**

It is thought that about 68% of all Year 12 students have their driver's licence by the time they leave high school. How large a sample would be needed to establish this to within a margin of error of 5% at the 95% confidence level?

$\hat{p} = 68\% = 0.68$

$z = 1.96 \quad (95\% \text{ CI})$

$\text{M of E} = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0.05 = 1.96\sqrt{\frac{0.68(0.32)}{n}}$

$n = 334.37$

$\therefore$ 335 students in sample.

✓ correct M of E using $\hat{p}$ & z

✓ correct n as decimal

✓ correct n rounding up

Question 6 **(3 marks)**

A random sample of 75 people were asked "Do you prefer AFL to soccer? From this a confidence interval $0.763 \le p \le 0.917$ was established at the x level.

How many people agreed with the question in the survey (*ie*: preferred AFL to soccer)

1. $\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{75}} = 0.763$

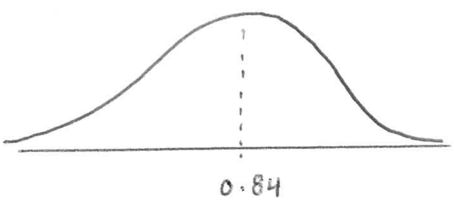
2. $\hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{75}} = 0.917$

use CAS to solve

$\hat{p} = 0.84$

$z = 1.819$

✓ correct equation for upper & lower boundary



$\hat{p} = \frac{\text{number in sample } x}{75}$

$0.84 = \frac{x}{75}$

$x = 63$

63 people agreed.

✓ use CAS to find $\hat{p}$

✓ correct number of people.

Question 7 **(9 marks)**

It is known that 30% of households in a large city own a bike.

a) Let X be the random variable that represents the number of bike-owning households chosen in a random sample of 60 households.

i) Describe the distribution of X and state its mean and standard deviation. (2 marks)

$\mu = np = 60 \times 0.3 = 18$

$\sigma = \sqrt{np(1-p)} = \sqrt{60(0.3)(0.7)} = 3.55$

$X \sim B(60, 0.3)$

✓ for distribution

✓ for μ AND σ

ii) Determine the probability that 21 or more bike-owning households will be chosen in a sample of 60. (1 mark)

$P(X \geq 21) = 0.2378$

✓ correct probability

b) A large number of random samples of 60 households are taken and each sample is used to calculate a point estimate for the proportion of bike-owning households in the city.

i) Describe the distribution of these sample proportions and state the mean and standard deviation of the distribution. (2 marks)

$\mu = \hat{p} = 0.3$

$\sigma = \sqrt{\frac{0.3(0.7)}{60}}$

$= 0.059$

$\hat{p} \sim N(0.3, 0.059^2)$

✓ correct distribution

✓ μ AND σ

ii) By providing mathematical evidence, show that the distribution in b)i) is appropriate to approximate the distribution in a). (2 marks)

$n = 60 > 30$

$np = 60 \times 0.3 = 18 > 10$

$n(1-p) = 60 \times 0.7 = 42 > 10$

$\therefore$ Appropriate to approximate a Normal Distribution.

✓ making 3 statements

✓ showing 3 statements to be true.

iii) Using the distribution in b)i), determine the probability that a particular sample of 60 households will have 21 or more bike-owning households. (2 marks)

$P\left(\hat{p} > \frac{21}{60}\right) = P(\hat{p} > 0.35) = 0.1990$

✓ correct $\hat{p}$

✓ correct probability

Question 8 **(9 marks)**

In a random sample of 50 people, 18 indicated that they had used the new airport train line sometime in the last year.

a) State the sample proportion, $\hat{p}$. (1 mark)

$$\hat{p} = \frac{18}{50}$$

✓ $\hat{p}$

b) Determine the sample standard deviation. (2 marks)

$$\sigma = \sqrt{\frac{0.36(0.64)}{50}}$$

$$= 0.068$$

✓ σ rule
✓ correct σ

c) Determine the 95% confidence interval for the population proportion p. (2 marks)

e Activity
or
C.A.S

$$0.227 \leq p \leq 0.493$$

✓ upper boundary
✓ lower boundary

d) Describe what happens to the confidence interval width if we increase our level of confidence to 99%. (1 mark)

Confidence interval width increases

✓ correct statement.

In a second sample of 50 people, 23 people were found to have used the new airport train line last year.

e) Using your previously calculated confidence interval, determine with reason if the result of the second sample is statistically different from that of the first. (3 marks)

$$\hat{p} = \frac{23}{50} = 0.46$$

This value of $\hat{p}$ is within the above 95% CI
∴ Second sample is not statistically different.

✓ $\hat{p}$ OR CI
✓ reason
✓ NOT statistically different.

OR

New CI : $0.322 \leq p \leq 0.598$
Similar overlap ∴ not statistically different.

Question 9 **(12 marks)**

Scientists have discovered that the leaves of a gum tree are normally distributed and have a mean length of 14.7 cm with standard deviation of 36 mm.

$\sigma = \cancel{}$ cm 3.6 cm

a) Determine the probability that a leaf selected from a gum tree has a length larger than 12.2 cm. (2 marks)

$X \sim N(14.7, \underset{3.6}{\cancel{0.36}}^2)$

$P(X > 12.2) = 0.7563$

✓ correct distribution

✓ correct propability

b) Determine the probability that a leaf selected from a gum tree has length between 12.2 cm and 14.7 cm if it is less than 15 cm. (2 marks)

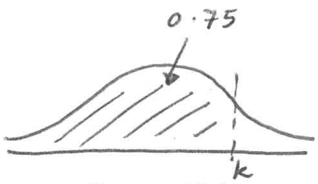
$P(12.2 \le X \le 14.7 \mid X < 15) = \dfrac{P(12.2 \le X \le 14.7)}{P(X < 15)}$

✓ correct conditional statement

$= \dfrac{0.2563}{0.5332} = 0.4807$

✓ correct probability (NOT left in fraction).

c) Determine the 0.75 quantile length for leaves of a gum tree. (2 marks)



$P(X < k) = 0.75$

✓ correct statement

$k = 17.13$ cm

✓ correct k.

d) 10 leaves are randomly selected from a gum tree to be further analysed. Determine the probability that at least half of these leaves have a mean length between 12.2 and 14.7 cm. (3 marks)

$Y \sim B(10, 0.2563)$

✓ correct distribution

$P(Y \ge 5) = 0.0857$

✓ correct probability statement

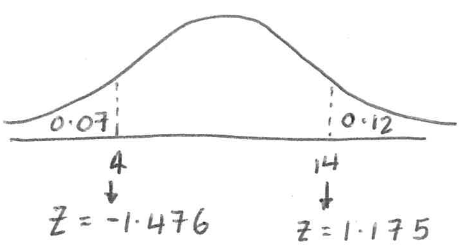
✓ correct probability.

A different type of tree, the Eucalyptus, has 7% of its leaves less than 4 cm and 12% of its leaves greater than 14 cm.

e) Determine the mean and standard deviation of the Eucalyptus leaf length. (Assume the length of the Eucalyptus leaves are normally distributed.) (3 marks)

$$\begin{cases} -1.476 = \dfrac{4 - \mu}{\sigma} \\ 1.175 = \dfrac{14 - \mu}{\sigma} \end{cases}$$

✓2 correct z-score statements



✓ μ

$\mu = 9.568$ cm ✓ σ

$\sigma = 3.772$ cm

END OF QUESTIONS